Probabilistic approach to reliability prediction for LWR fuel rods

Misfeldt, I.

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Probabilistic Approach to Reliability Predictions for LWR Fuel Rods

by

Ib Misfeldt

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Abstract

It is generally recognized that structural problems are often non-deterministic. This is also the case for power reactor fuel. The stochastic behaviour of such fuel has been clearly demonstrated from numerous test reactor experiments, as well as from power reactor experience.

Any attempt to calculate the reliability of nuclear fuel thus leads to a probabilistic approach, as outlined in this paper.

The first part of the fuel reliability prediction is the calculation of the fuel state (distributions of temperature, stress, strain, etc., in pellet and clad) as a function of time. This part is implemented as a computer code, FRP.

This paper summarises the theory behind FPP and presents an example of a reliability calculation.

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CONTENTS

1. INTRODUCTION

It is generally recognized that structural safety should be expressed in probabilistic rather than in deterministic terms. Failure should be interpreted with respect to some predefined limit state, a failure criterion. For a fuel rod, the failure criterion could be cracking of the cladding, bowing of the rod, or just exceeding a design limit such as maximum allowed clad strain. Therefore, depending on the failure criterion under consideration, the concept of failure probability is applicable to both the safety and the performance of structures.

In principle, the probability of failure has an absolute meaning, the likelihood of the occurrence of some specified unfavourable state. For many practical purposes, however, it is sufficient to interpret the structural safety in a relative sense quantified by a reliability index. In this sense the reliability index serves as a common and logical basis for the evaluation of the performance and safety of components and systems.

In reactor fuel technology the deterministic design and test procedure has proved to be inadequate. Though limitations concerning reactor operation are severe, failures still occur and they can cause unscheduled shut-downs of plants with serious economic consequences. The exact reasons for these failures cannot always be evaluated. Therefore, the safety margins for the fuel are increased either through a "stronger" design, or by more severe limitations to reactor operation. Tolerances relating to the fuel are decreased to very low levels, though no clear connection with fuel reliability has yet been proven. The consequence is increased fuel costs and reduced flexibility in the operation of power plants.

This situation can be improved through a probabilistic approach. Knowing the failure mechanisms, a reliability index for different fuel designs can be computed for given operational conditions. This reliability index will not necessarily agree with the safety margins as calculated by deterministic models, but it offers a quantitative basis for comparison of different designs. The sensitivity of the safety index to design variables, tolerances and material specifications gives an indication of how the reliability of the design can be improved most economically.

When preparing and evaluating experiments it is important to know the expected distributions of the results and their sensitivity to uncontrolled parameters and tolerances on the specifications. From numerous ramp experiments, it is obvious that the distribution of the experimental results is important. Profit from these experiments could be considerably increased by introducing probabilistic considerations in the planning as well as in the evaluation of the tests.

Reliability calculations could also be beneficial for operating power plants.

If a quantitative calculation of the probability of failures for given operating conditions could be made, then utilities would have a good basis for decisions aimed at optimal plant operation.

The approach to reliability calculations on fuel outlined in this paper is mainly aimed at the evaluation of different designs and the analysis of experimental results.

Two standard methods are utilized in FRP for the calculation of the fuel state (distributions of temperatures, strains, etc., in pellet and cladding) as functions of time. They are the Monte Carlo method[1)] and the method of partial derivatives[2)]. Only the influence of design and material parameters is considered, since this is important when comparing designs and analyzing experimental data.

At the present state of the model development only simple failure criteria are considered, the failure probability being computed by hand based on the calculated fuel state data.

2. EVALUATION OF CONCEPTS

Consider a case where a structure with a stochastic resistance R is subject to a stochastic load S. Let the failure criterion be $R<S$. If the probability density functions, $f_R(r)$ and $f_S(s)$, of R and S are known, the probability of failure is $P(R<S)$

$$P(\text{failure}) = P(R<S)$$

$$= \int_0^{\infty} \left[\int_0^{S} f_R(r)dr \right] f_S \, ds$$

$$= \int_0^{\infty} F_R(s) f_S(s) ds \tag{1}$$

see fig. 1

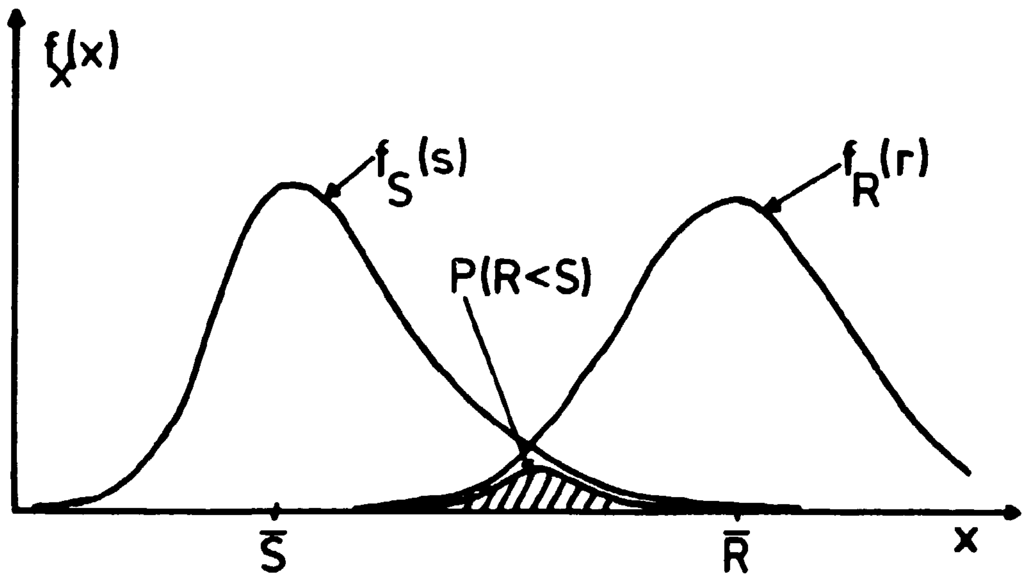

Fig. 1. Probability of failure

In general, the probability of failure can be associated with a failure region defined by

$F(\underline{X}) < 0$,

where $\underline{X}$ is a vector of random variables. For the previous example $F(R,S) < 0$ and $F(R,S) = R-S$. The probability of failure is then $P(F(\underline{X})<0)$.

Since $F(\underline{X})$ is often a complicated function of random variables, the closed form analytical solution of $P(F(\underline{X})<0)$ is seldom possible, therefore approximations are used to estimate $P(F(\underline{X})<0)$.

Monte Carlo simulation consists of computing a number of values of $F(\underline{X})$, each based on a new random sample from $\underline{X}$. If the number of samples is large, the calculated values of $F(\underline{x}_i)$ form a good approximation to $F(\underline{X})$. By this method the pdf (probability density function) of $F(\underline{X})$ can be approximated arbitrarily well, regardless of the form of the pdf's for the X_i's, but no information is obtained regarding the sensitivity of $F(\underline{X})$ to the individual X_i's.

By the method of partial derivatives[2)] an approximation to the mean and standard deviation of $F(\underline{X})$ is calculated by expanding $F(\underline{X})$ about $F(\underline{\bar{X}})$ the point where all variables X_i assume their mean value $\bar{X}_i$). Retaining only the lowest non-zero terms, the final expression for $\overline{F(\underline{X})}$ and $s_{F(\underline{X})}$ is

$$\overline{F(\underline{X})} = F(\underline{\bar{X}}) \tag{2}$$

$$s_{F(\underline{X})} = \Big[\sum_{i=1}^{n} \Big(\frac{\partial F(\underline{\bar{X}})}{\partial X_i} s_{X_i} \Big)^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \Big(\frac{\partial F(\underline{\bar{X}})}{\partial X_i} \Big) \Big(\frac{\partial F(\underline{\bar{X}})}{\partial X_j} \Big) \mathrm{cov}(X_i, X_j) \Big]^{\frac{1}{2}} \tag{3}$$

where $\mathrm{cov}(X_i, X_j)$ is the covariance between the variables X_i and X_j. If X_i and X_j are independent, $\mathrm{cov}(X_i, X_j) = 0$.

The method of partial derivatives directly gives the contribution to $s_{F(X)}$ from the individual X_i's, as well as the sensitivity of F to the X_i's. In design calculations this is especially important, since it gives a basis for deciding where the tolerances are of importance and where the design can be improved.

The failure probability $P(F(\underline{X})<0)$ can only be calculated if a distribution function for $F(\underline{X})$ is assumed.

In many applications the exact reliability of the structure is not needed, or it cannot be evaluated, because the distributions of the independent variables, X_i's, are unknown. In these applications it is still possible to calculate a quantitative measure of the reliability, a reliability index. The reliability index, β, is defined[3)] as the distance to the failure region when all variables are measured in standard deviation units.

For a structure, let S and R be random variables with mean $\bar{S}$, $\bar{R}$, standard deviation s_S, s_R and P(failure = $P(F(S,R)<0)$, where $F(S,R) = R-S$. The reliability index for this structure, β, is

$$\beta = \frac{\bar{R}-\bar{S}}{\left(s_R^2+s_S^2\right)^{\frac{1}{2}}}$$

independent of the distribution of S and R. If S and R are normally distributed, the probability of failure is

$$P(R-S<0) = 1-\emptyset(\beta)$$

where $\emptyset(X)$ is the cumulative standardized normal distribution function.

The reliability index is the logical basis for comparing different designs, especially if the distribution of the variables considered is not known. This is normally the case where high reliabilities are considered, since it is the extreme tails of the pdf that determine the reliability and the available data are limited to the central ranges of the variables.

3. THE COMPUTER PROGRAM FRP

The foregoing concepts are implemented as the computer program FRP. The program requires as input the same design and power history data as the deterministic fuel code FFRS[4)] and, in addition, information regarding the assumed distribution for the design parameters. Default values for the material parameters are assumed; they are based on ref. 5. It should be noticed that the distribution for the material data is based on literature data from various sources, therefore the distributions are not typical of a single case but include the variability originating from fabrication differences. A typical example is Zircaloy creep, where part of the uncertainty is due to the different texture in the experimental data examined. Therefore, if a certain batch of cladding tubes is considered, the variation is much lower, but the mean value for the batch does not need to coincide with the mean value for the creep equation in FRP.

The Monte Carlo samples as well as the numerical approximations to the partial derivatives are calculated utilizing the fuel performance code FFRS.

4. EXAMPLE

An experiment, pin PA21-3, from the Danish irradiation programme was analyzed with FRP. The experiment is described in refs. 4 and 6. The fuel pin was irradiated to 20000 MWd/tUO_2 in the Halden boiling-water reactor and ramp-tested in the DR 3 reactor at Risø. The Halden irradiation, the ramp test, the stresses and strains calculated by FFRS[4)], and the axial power shapes are shown on figures 2-6. The pre- and post-ramp profilometer traces, fig. 6, show small midpellet diameter increases and some ridge formation. The ridge formation and the midpellet diameter increases vary irregularly along the pin, with a maximum midpellet increase of 30 μm, which can be characterized as "de-ovalization".

The deterministic analysis of the experiment, figures 4 and 5, indicates virtually no diameter increase. A simple sensitivity analysis, a calculation with 40% reduced gap, indicates large sensitivity to the "as-fabricated" gap.

The experiment was analyzed by the method of partial derivatives and the Monte Carlo method. The results for the midpellet strain during the ramp are shown in table I. The distribution of the strain is shown in figure 8. As seen in table I and on figure 8, the values calculated by FRP are in good agreement with the experimental observations.

The influence of design and material parameters on the variance of the strain is shown in figure 9. This figure also shows the contribution to the variance of the strain, calculated under the assumption that the Halden irradiation was performed at normal BWR conditions (70 atm and 295°C). The pronounced difference between the two sets of operating conditions demonstrates that great care is necessary when utilizing tests performed at conditions different from normal reactor operating conditions.

5. FAILURE PROBABILITY

The ultimate goal of the analysis is to calculate the probability of failure for the fuel pin considered. The example was therefore analyzed under the assumption that the failure in the ramp test is correlated with the strain in the cladding during the ramp.

The failure criterion could be the exceeding of either max. uniform strain or total elongation. From 19 experimental values[7)], the following distributions for max. uniform strain and total elongation for irradiated cladding are calculated,

R_u = max. uniform strain = N(0.21%, 0.04%)*

R_t = total elongation = N(1.7%, 0.35%)*

When considering average strains as calculated by FFRS, it seems reasonable to use max. uniform strain as the failure criterion.

The strain during the ramp as calculated by the Monte Carlo method is

$\bar{\varepsilon} = 0.029$ and $s_\varepsilon = 0.045$

The reliability index, β, with the failure criterion P(failure) = $P(\varepsilon > R_u)$ is

* N(a,b) means normally distributed with mean value a and standard deviation b.

$$\beta = \frac{0.21 - 0.029}{(0.04^2+0.045^2)^{\frac{1}{2}}} = 3.0$$

This indicates a rather low probability of failure.
If a distribution is assumed for ε, the probability of failure can be calculated from (1). Assuming normal distribution for ε gives

$$P(\text{failure}) = P(\varepsilon > R_u) = 1 - 0.9987 = 1.4 \times 10^{-3}$$

The Weibull distribution seems to fit the upper part of the distribution for ε quite well, as shown on figure 10. Numerical evaluation of the convolution integral (1) yields

$$P(\text{failure}) = P(\varepsilon > R_u) = 9 \times 10^{-3}.$$

In table II the calculated probability of failure is shown for both failure criteria. The table also includes calculations based on the assumption that the maximum strain (at ridges) is twice the midpellet strain, which is typical for PA21-3. The failure probabilities are calculated per pellet, the total failure probability for the fuel pin is estimated by assuming 15 equal pellets. This gives a failure probability of 13% for the midpellet strain and 52% for the maximum strain, under the assumption that the strain is Weibull-distributed.

This calculated failure probability can quite well explain the observed failure for pin PA21-3.

6. CONCLUSION

A program for the calculation of fuel reliability is presented.

The first part of the calculations, the calculation of the fuel state as a function of time, is implemented as a computer code FRP.

An irradiation experiment was analyzed by means of FRP; the distribution on the midpellets strain, as calculated by FRP, agrees well with the distribution observed in the experiment.

The failure probability was evaluated assuming the exceeding of the uniform elongation to be the failure criterion. There is reasonable agreement between the calculated probabilities of failure and the fact that the pin failed.

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TABLE 1

Midpellet diameter increases in ramp test PA21-3

Method	Diameter increase	
	Mean, μm	Standard deviation, μm
<u>Experimental</u>		
120°, 15 pellets	4	5
Max. experimental	30*	-
Deterministic calculation, FFRS	1.8	-
Partial derivatives	1.8	4.9
Monte Carlo Simulations	4.0	6.3

* due to "de-ovalization", max. at 120° = 15 μm.

TABLE II

Calculated probability of failure for PA21-3

Failure criterion	reliability index, β	failure probability per pellet	failure probability for 15 pellets
$P(\varepsilon > R_u)$ ε normal distributed	3.0	1.4×10^{-3}	2.1%
$P(\varepsilon > R_u)$ ε Weibull distributed	3.0	9×10^{-3}	13%
$P(2\varepsilon > R_u)$ ε normal distributed	1.5	0.067	65%
$P(2\varepsilon > R_u)$ ε Weibull distributed	1.5	0.048	52%
$P(\varepsilon > R_t)$ ε normal distributed	4.76	$<4\times10^{-6}$	$<6\times10^{-5}$
$P(\varepsilon > R_t)$ ε Weibull distributed	4.76	$<2\times10^{-5}$	$<3\times10^{-4}$
$P(2\varepsilon > R_t)$ ε normal distributed	4.54	$<4\times10^{-6}$	$<6\times10^{-5}$
$P(2\varepsilon > R_t)$ ε Weibull distributed	4.54	$<3\times10^{-5}$	$<5\times10^{-4}$

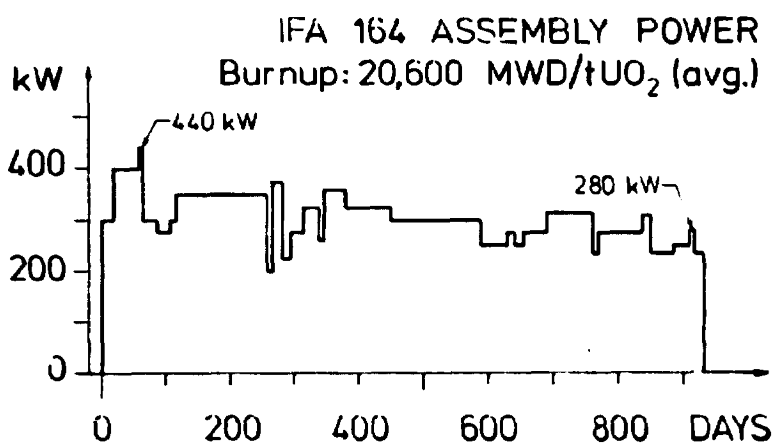

Fig. 2. Assembly power during the Halden irradiation

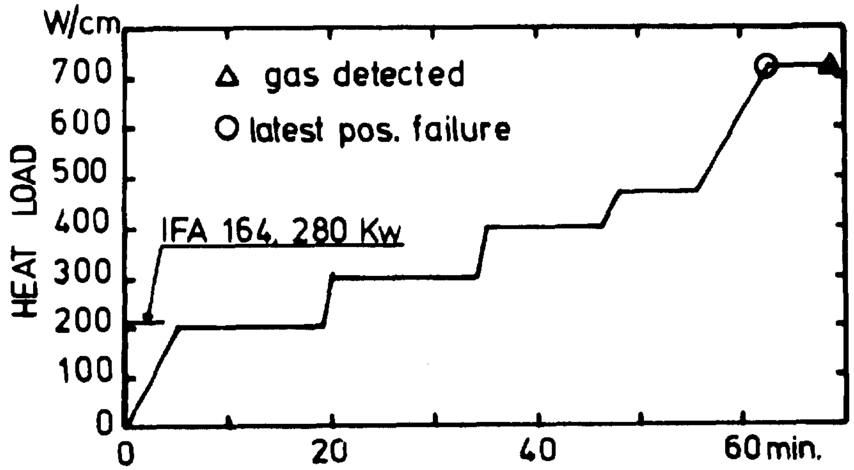

Fig. 3. Power ramp for PA21-3.

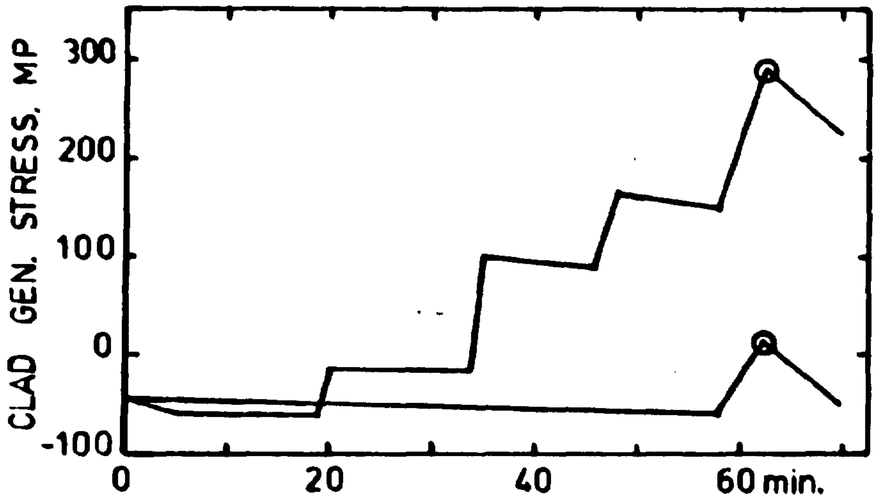

Fig. 4. Generalized clad stress of PA21-3.

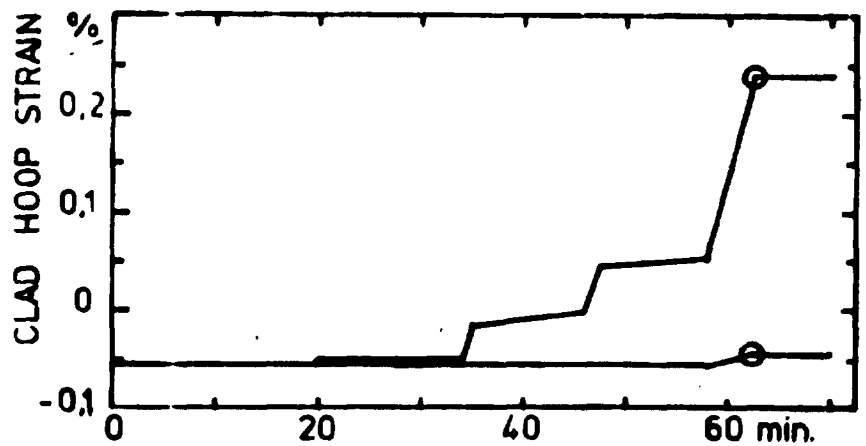

Fig. 5. Clad hoop strain of PA21-3.

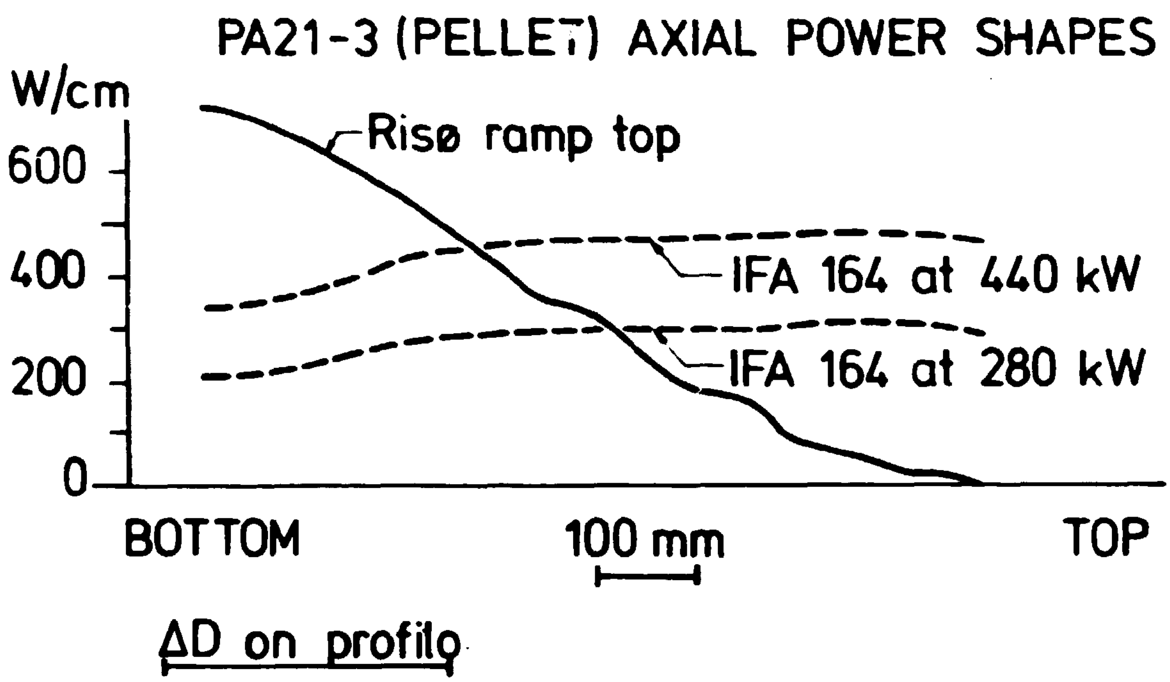

Fig. 6. Axial power shapes for PA21-3.

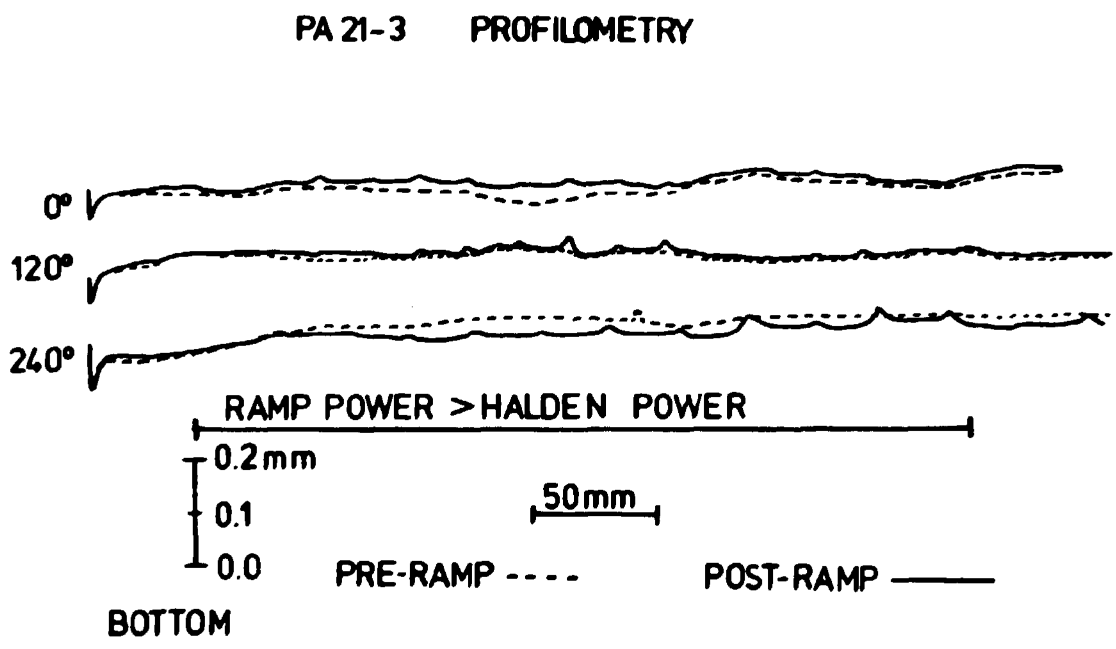

Fig. 7. Profilometer traces for PA21-3. The curves are manual reproduced and should not be used for measurements.

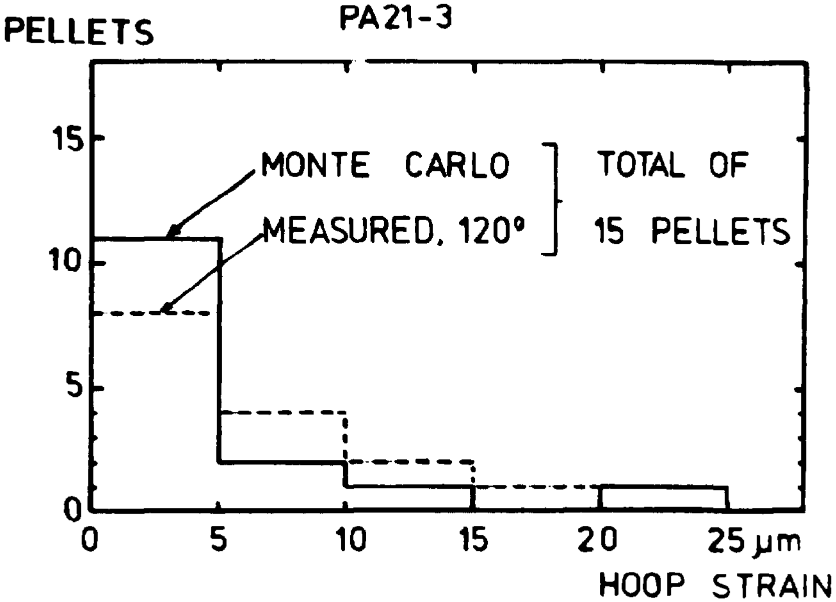

Fig. 8. PA21-3. Distribution of the midpellet diameter increase.

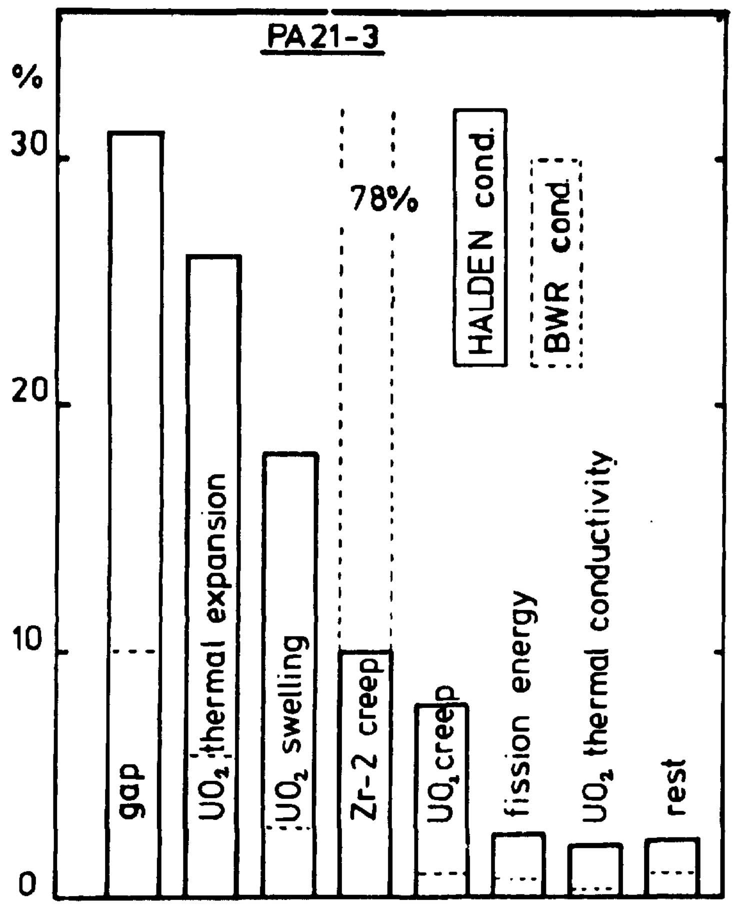

Fig. 9. PA21-3. Contribution to the variance on the hoop strain.

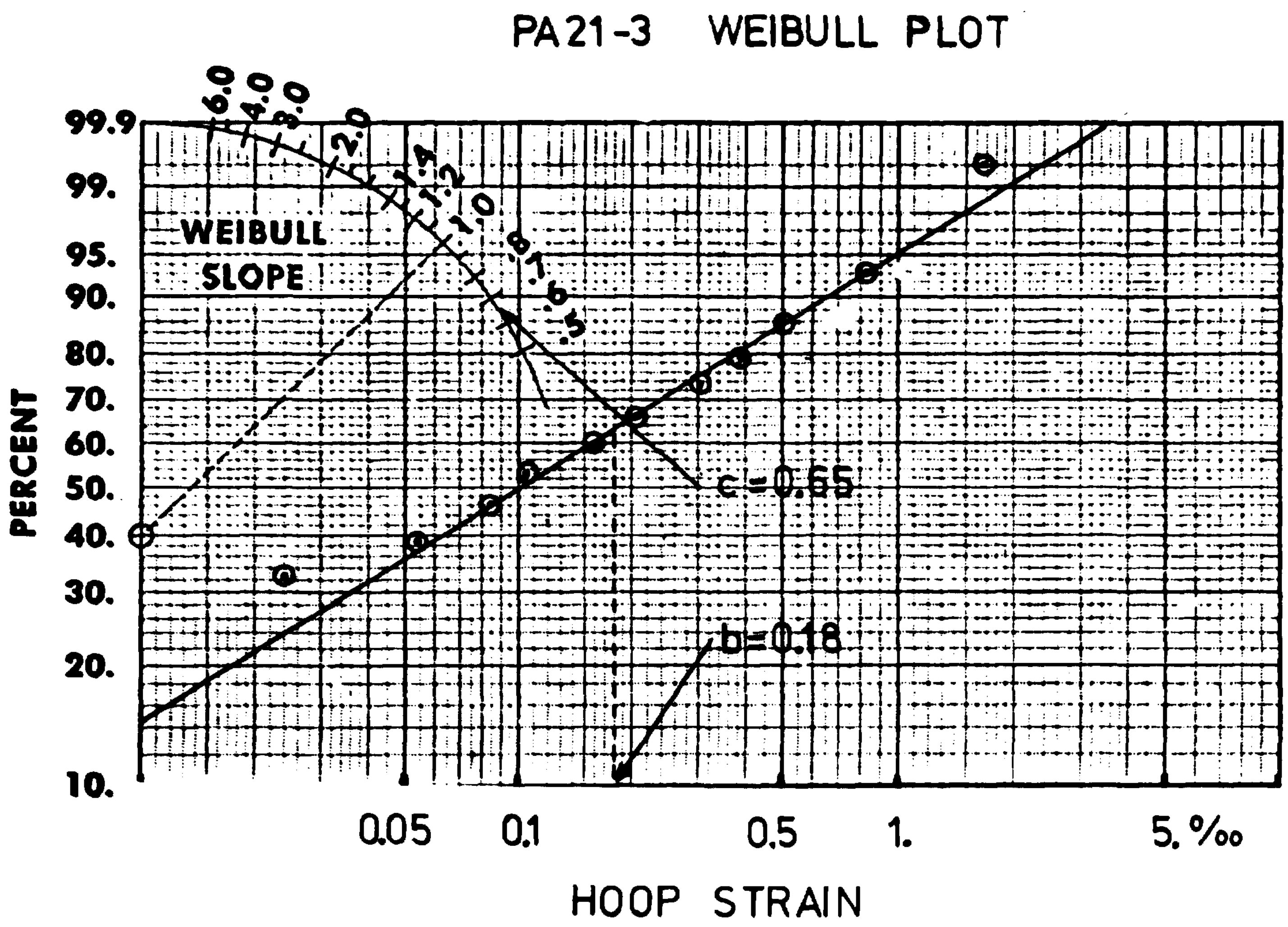

Fig. 9. PA21-3. Weibull plot of the calculated (by Monte Carlo simulation) cumulative distribution function for the midpellet strain.